

Math 2J  
Quiz 1

Key

1a. What is the augmented matrix corresponding to this system of equations?

$$\begin{array}{rclcl} x_1 & + & x_2 & - & x_3 & = & 1 \\ 2x_1 & - & x_2 & - & 2x_3 & = & 5 \\ x_1 & + & 4x_2 & - & 1x_3 & = & -2 \end{array} \quad \left[ \begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 2 & -1 & -2 & 5 \\ 1 & 4 & -1 & -2 \end{array} \right]$$

1b. Use Gaussian elimination to put this augmented matrix in reduced row echelon form.

$$\begin{array}{l} \left[ \begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 2 & -1 & -2 & 5 \\ 1 & 4 & -1 & -2 \end{array} \right] \xrightarrow[\substack{R_2' = R_2 - 2R_1 \\ R_3' = R_3 - R_1}]{\phantom{R_2' = R_2 - 2R_1}} \left[ \begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & -3 & 0 & 3 \\ 0 & 3 & 0 & -3 \end{array} \right] \xrightarrow{R_3'' = R_3' + R_2'} \\ \left[ \begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & -3 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{R_2'' = -\frac{1}{3}R_2''} \left[ \begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{R_1''' = R_1'' - R_2''} \left[ \begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{array} \right] \end{array}$$

1c. Solve the system of equations using your result from 1b.

The second row gives us  $x_2 = -1$ . The first row gives us  $x_1 - x_3 = 2$ .

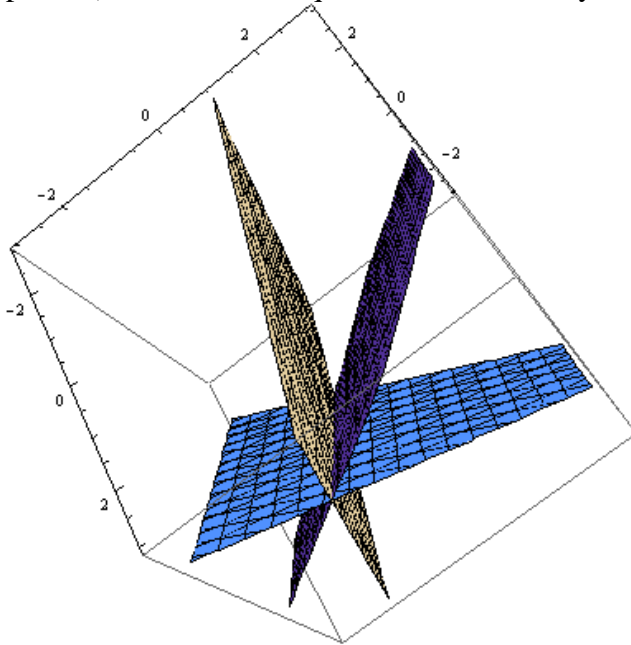
There are many different-looking solutions (that actually specify the same solution set), depending on what you choose as your free variable. Here are the two ways of writing the solutions based on the methods we discussed in class:

$x_3$  is free:  $x_1 = 2 + x_3$ . If  $\alpha$  is any real number then our answers are all of the form  $(2 + \alpha, -1, \alpha)$ .

$x_1$  is free:  $x_3 = x_1 - 2$ . If  $\alpha$  is any real number then our answers are all of the form  $(\alpha, -1, \alpha - 2)$ .

1d. What does your solution imply about how the planes in 1a interact?

The planes in 1a intersect along a line. Here's a sketch of the planes to illustrate the point: (You were not required to sketch the system of equations on your quiz!)



2. Multiply the following matrices if possible. If it is not possible, explain why.

$$\begin{bmatrix} 1 & 2 \\ -1 & 3 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -1 & 3 \\ -2 & 0 \end{bmatrix}_{3 \times 2} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2 \times 2} = \begin{bmatrix} [1 \ 2] \cdot \begin{bmatrix} 1 \\ 3 \end{bmatrix} & [1 \ 2] \cdot \begin{bmatrix} 2 \\ 4 \end{bmatrix} \\ [-1 \ 3] \cdot \begin{bmatrix} 1 \\ 3 \end{bmatrix} & [-1 \ 3] \cdot \begin{bmatrix} 2 \\ 4 \end{bmatrix} \\ [-2 \ 0] \cdot \begin{bmatrix} 1 \\ 3 \end{bmatrix} & [-2 \ 0] \cdot \begin{bmatrix} 2 \\ 4 \end{bmatrix} \end{bmatrix}_{3 \times 2} = \begin{bmatrix} 7 & 10 \\ 8 & 10 \\ -2 & -4 \end{bmatrix}$$